

Quantum Mechanical Localization Effects for Bose-Einstein Correlations

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For a set of N identical massive boson wavepackets with optimal initial quantum mechanical localization, we calculate the Hanbury-Brown/Twiss (HBT) two-particle correlation function. Our result provides an algorithm for calculating one-particle spectra and two-particle correlations from an arbitrary phase space occupation $(\mathbf{q}_i, \mathbf{p}_i, t_i)_{i=1,N}$ as e.g. returned by event generators. It is a microscopic derivation of the result of the coherent state formalism, providing explicit finite multiplicity corrections. Both the one- and two-particle spectra depend explicitly on the initial wavepacket width σ which parametrizes the quantum mechanical wavepacket localization. They provide upper and lower bounds which suggest that a realistic value for σ has the order of the Compton wavelength.

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Two-particle correlations $C(\mathbf{Q}, \mathbf{K})$ of identical particles are the only known observables giving access to the space-time structure of the particle emitting source in heavy ion collisions. Their interpretation is based on the result of the coherent state formalism [1,2] which reads in the plane wave approximation for a large number of sources

$$C(\mathbf{Q}, \mathbf{K}) = 1 + \frac{|\int d^4x S(x, K) e^{ix \cdot Q}|^2}{\int d^4x S(x, P_1) \int d^4y S(y, P_2)}, \quad (1a)$$

$$Q = P_1 - P_2, \quad K = \frac{1}{2}(P_1 + P_2). \quad (1b)$$

In this setting, an Hanbury-Brown/Twiss (HBT) interferometric analysis aims at extracting from the correlator $C(\mathbf{Q}, \mathbf{K})$ as much information as possible about the space-time emission function $S(x, K)$. Since this emission function cannot be reconstructed unambiguously from $C(\mathbf{Q}, \mathbf{K})$ [3], (1) is mainly used in the study of model emission functions $S(x, K)$. These studies have clarified to a considerable extent the question which geometrical and dynamical source characteristics are reflected in which particular momentum dependencies of the correlator (cf. [3] and refs. therein). A comparison with measured correlations then allows to constrain the class of source models consistent with data.

Microscopic event generators are one important tool to generate model emission functions. Here, we do not discuss in how far existing event generators (e.g. RQMD [4], VENUS [5], ARC [6]) provide an internally consistent calculation of the phase space distribution. None of them propagates (anti)-symmetrized N-particle

states from first principles, and the resulting difficulties in calculating 2-particle correlations have been discussed recently in great detail [7]. The typical event generator output is a set Σ of phase space points at given times $z_i = (\mathbf{q}_i, \mathbf{p}_i, t_i)$ which one associates with the “points of last interactions”. However, the Heisenberg uncertainty principle allows to interpret the z_i only as mean positions of boson wave packets. To specify the localization of these wavepackets in phase space, at least one additional parameter is needed, e.g. the initial spatial wavepacket width σ . Irrespective of how the phase space occupation has been obtained, we shall take the set Σ and the width σ as initial condition for the present investigation: Σ and σ define the boson emitting source. For notational simplicity, we restrict our discussion to one particle species, negative pions, say.

The problem in associating an emission function $S(x, K)$ to the distribution Σ is that Σ is a *discrete* phase space distribution of *on-shell* particles. In contrast, the emission function $S(x, K)$ of the coherent state formalism is a *continuous* distribution which allows for *off-shell* momenta K . Often, one circumvents this problem by approximating the Pratt algorithm [8] by an ad hoc prescription: each particle pair (i, j) is weighted with a probability ρ_{ij} , q_i being 4-vectors $q_i = (t_i, \mathbf{q}_i)$,

$$C(\Delta\mathbf{Q}, \Delta\mathbf{K}) = \frac{1}{N(\Delta\mathbf{Q}, \Delta\mathbf{K})} \sum_{(i,j)} \rho_{ij}, \quad (2a)$$

$$\rho_{ij} = 1 + \cos((p_i - p_j) \cdot (q_i - q_j)). \quad (2b)$$

Here, $C(\Delta\mathbf{Q}, \Delta\mathbf{K})$ denotes the 2-particle correlator for pairs whose relative and average pair momenta $\mathbf{p}_i - \mathbf{p}_j$, $\frac{1}{2}(\mathbf{p}_i + \mathbf{p}_j)$ lie in the bin $\Delta\mathbf{Q}, \Delta\mathbf{K}$. $N(\Delta\mathbf{Q}, \Delta\mathbf{K})$ is the corresponding number of particle pairs. A tentative argument to justify the prescription (2) is that ρ_{ij} coincides with the formal Born probability density $\Psi^* \Psi$ of the Bose-Einstein symmetrized 2-particle plane wave

$$\rho_{ij} = \Psi^*(q_i, q_j, p_i, p_j) \Psi(q_i, q_j, p_i, p_j), \quad (3a)$$

$$\Psi(q_i, q_j, p_i, p_j) = \frac{1}{\sqrt{2}} (e^{ip_i q_i + ip_j q_j} + e^{ip_j q_i + ip_i q_j}). \quad (3b)$$

However, the prescription (2) based on the ansatz (3) is inconsistent [9] with the result (1) of the coherent state formalism: The correlator in (1) is always larger than unity [3]. In contrast, the expression (2) can drop below unity in the region of sufficiently large relative momenta [9]. Also, the prescription (2) is difficult to reconcile with quantum mechanical localization requirements since the

plane wave (3b) cannot be an eigenstate for both the position and momentum operator.

In what follows, we take the quantum mechanical localization of bosons into account by associating to the phase space emission points z_i free Gaussian wavepackets of initial spatial width σ , [10,11]

$$f_{z_i}^{(\sigma)}(\mathbf{X}, t) = (\pi\sigma^2)^{-\frac{3}{4}} \left(\frac{\sigma^2}{\sigma_i^2(t)} \right)^{\frac{3}{2}} \exp(i\mathbf{p}_i\mathbf{X} - i\mathcal{E}_i t) \times \exp\left(-\frac{1}{2\sigma_i^2(t)} (\mathbf{X} - \mathbf{q}_i(t))^2\right) \quad (4a)$$

$$\mathbf{q}_i(t) = \mathbf{q}_i + \frac{\mathbf{p}_i}{m}(t - t_i), \quad \mathcal{E}_i = \frac{\mathbf{p}_i^2}{2m}, \quad (4b)$$

$$\sigma_i^2(t) = \sigma^2 + i \frac{(t - t_i)}{m}. \quad (4c)$$

This one boson state (4a) is optimally localized around $(\mathbf{q}_i, \mathbf{p}_i)$ in the sense that it saturates the Heisenberg uncertainty relation $\Delta\mathbf{x} \cdot \Delta\mathbf{p}_x = 1$, with $\Delta x_i = \sigma$ at time $t = t_i$. The time evolution of (4) is the free unperturbed evolution determined by the Hamiltonian $H_0 = \frac{\Delta}{2m}$, Δ being the Laplacian. Since the i -th and j -th boson are identical, we associate to the two emission points z_i and z_j the symmetrized two boson wave function $\Phi_{ij}(\mathbf{X}, \mathbf{Y}, t)$ (the normalization factor is omitted and plays no role in what follows)

$$\Phi_{ij}(\mathbf{X}, \mathbf{Y}, t) = f_{z_i}^{(\sigma)}(\mathbf{X}, t) f_{z_j}^{(\sigma)}(\mathbf{Y}, t) + f_{z_i}^{(\sigma)}(\mathbf{Y}, t) f_{z_j}^{(\sigma)}(\mathbf{X}, t). \quad (5)$$

We now derive an algorithm for calculating one-particle spectra $\nu(\mathbf{P})$ and two-particle correlations $C(\mathbf{P}_1, \mathbf{P}_2)$ from an arbitrary initial phase space distribution Σ of best localized boson wavepackets $f_{z_i}^{(\sigma)}$. Our first step is to calculate for two identical bosons the detection probability at time t at the positions \mathbf{X} and \mathbf{Y} with momenta $\mathbf{P}_1, \mathbf{P}_2$ respectively. This is given by the two-particle Wigner phase space density [12]

$$W_{ij}(\mathbf{X}, \mathbf{Y}, \mathbf{P}_1, \mathbf{P}_2, t) = \Phi_{ij}(\mathbf{X}, \mathbf{Y}, t) (2\pi)^6 \delta^{(3)}(\mathbf{P}_1 - \hat{\mathbf{P}}_1) \times \delta^{(3)}(\mathbf{P}_2 - \hat{\mathbf{P}}_2) \Phi_{ij}^*(\mathbf{X}, \mathbf{Y}, t) \\ = \int d^3\mathbf{X}_1 d^3\mathbf{Y}_1 \Phi(\mathbf{X} + \frac{\mathbf{X}_1}{2}, \mathbf{Y} + \frac{\mathbf{Y}_1}{2}, t) e^{i\mathbf{P}_1\mathbf{X}_1} \times e^{i\mathbf{P}_2\mathbf{Y}_1} \Phi^*(\mathbf{X} - \frac{\mathbf{X}_1}{2}, \mathbf{Y} - \frac{\mathbf{Y}_1}{2}, t). \quad (6)$$

The corresponding probability to detect these bosons with momenta \mathbf{P}_1 and \mathbf{P}_2 irrespective of their position is

$$P_{ij}(\mathbf{P}_1, \mathbf{P}_2) = \int d^3\mathbf{X} d^3\mathbf{Y} W_{ij}(\mathbf{X}, \mathbf{Y}, \mathbf{P}_1, \mathbf{P}_2, t) \\ = w_i(\mathbf{P}_1, \mathbf{P}_1) w_j(\mathbf{P}_2, \mathbf{P}_2) + w_i(\mathbf{P}_2, \mathbf{P}_2) w_j(\mathbf{P}_1, \mathbf{P}_1) \\ + 2 w_i(\mathbf{P}_1, \mathbf{P}_2) w_j(\mathbf{P}_1, \mathbf{P}_2) \times \cos((q_i - q_j) \cdot (\mathbf{P}_1 - \mathbf{P}_2)), \quad (7a)$$

$$w_i(\mathbf{P}_1, \mathbf{P}_2) = e^{-\frac{\sigma^2}{4}(\mathbf{P}_1 - \mathbf{P}_2)^2} s_i(\mathbf{K}), \quad (7b)$$

$$s_i(\mathbf{K}) = 2^3 (\pi\sigma^2)^{\frac{3}{2}} e^{-\sigma^2(\mathbf{p}_i - \mathbf{K})^2}. \quad (7c)$$

Here, P_i denotes 4-vectors $P_i = (\frac{1}{2m}\mathbf{P}_i^2, \mathbf{P}_i)$. We note that P_{ij} is independent of the detection time t , i.e., only the correlations which exist already at emission are measured at time t in the detector. This t -independence is a consequence of the free time evolution; it is lost if final state interactions are included in the evolution of the wavepackets (4). Neglecting higher order symmetrizations, we define the (unnormalized) two pion correlation $R(\mathbf{P}_1, \mathbf{P}_2)$ for a set of N phase space points z_i by summing the probabilities P_{ij} over all $\frac{1}{2}N(N-1)$ pairs (i, j)

$$R(\mathbf{P}_1, \mathbf{P}_2) = \sum_{(i,j)} P_{ij}(\mathbf{P}_1, \mathbf{P}_2) \\ = \nu(\mathbf{P}_1) \nu(\mathbf{P}_2) - 2 T_c(\mathbf{P}_1, \mathbf{P}_2) \\ + \left| \sum_{i=1}^N w_i(\mathbf{P}_1, \mathbf{P}_2) e^{it_i(E_1 - E_2) - i\mathbf{q}_i(\mathbf{P}_1 - \mathbf{P}_2)} \right|^2, \quad (8a)$$

$$\nu(\mathbf{P}) = \sum_{i=1}^N s_i(\mathbf{P}). \quad (8b)$$

Here, $s_i(\mathbf{P})$ is the one-particle probability (7c) that a boson in the state $f_{z_i}^{(\sigma)}$ is detected with momentum \mathbf{P} . Accordingly, $\nu(\mathbf{P})$ is the one-particle spectrum of the distribution Σ with spatial localization σ . The contribution T_c to $R(\mathbf{P}_1, \mathbf{P}_2)$ corrects for the fact that the sums in the other two terms of (8a) include the N identical pairs (i, i) which are not present in $R(\mathbf{P}_1, \mathbf{P}_2)$,

$$T_c(\mathbf{P}_1, \mathbf{P}_2) = \sum_{i=1}^N s_i(\mathbf{P}_1) s_i(\mathbf{P}_2). \quad (9)$$

To obtain a normalized 2-particle correlation $C(\mathbf{P}_1, \mathbf{P}_2)$, we choose the normalization

$$N(\mathbf{P}_1, \mathbf{P}_2) = \nu(\mathbf{P}_1) \nu(\mathbf{P}_2) - T_c(\mathbf{P}_1, \mathbf{P}_2), \quad (10a)$$

$$C(\mathbf{P}_1, \mathbf{P}_2) = \frac{R(\mathbf{P}_1, \mathbf{P}_2)}{N(\mathbf{P}_1, \mathbf{P}_2)}. \quad (10b)$$

This choice is motivated by the experimental praxis of “normalization by mixed pairs”: An uncorrelated (mixed) pair is described by an unsymmetrized product state

$$\Phi_{ij}^{\text{uncorr}}(\mathbf{X}, \mathbf{Y}, t) = f_{z_i}^{(\sigma)}(\mathbf{X}, t) f_{z_j}^{(\sigma)}(\mathbf{Y}, t), \quad (11)$$

for which the two particle Wigner phase space density and the corresponding detection probability $P_{ij}^{\text{uncorr}}(\mathbf{P}_1, \mathbf{P}_2)$ can be calculated according to (6). Taking both distinguishable states $\Phi_{ij}^{\text{uncorr}}$ and $\Phi_{ji}^{\text{uncorr}}$ into account, and summing over all pairs (i, j) , we find

$$N(\mathbf{P}_1, \mathbf{P}_2) = \sum_{(i,j)} P_{ij}^{\text{uncorr}}(\mathbf{P}_1, \mathbf{P}_2). \quad (12)$$

Hence, the normalization (10a) is the 2-particle detection probability for uncorrelated pairs. The correlator reads

$$C(\mathbf{P}_1, \mathbf{P}_2) = 1 + \frac{\left| \sum_{i=1}^N w_i(\mathbf{P}_1, \mathbf{P}_2) e^{iq_i Q} \right|^2 - T_c}{\nu(\mathbf{P}_1) \nu(\mathbf{P}_2) - T_c}. \quad (13)$$

In contrast to (2), this is a continuous function of the measured momenta P_1, P_2 , i.e., no binning of the correlator is required. Note that the normalization (10a) ensures that the correlator (13) is always smaller than 2 and equals 2 for $\mathbf{P}_1 - \mathbf{P}_2 = 0$. (This follows from the fact that the sum of the first two terms in (7a) is always larger than the third one.) For any boson source, defined by an arbitrary phase space distribution Σ and a spatial wavepacket width σ , Eq. (13) provides an algorithm of how to calculate the 2-particle correlator, using (7b), (7c), (8b) and (9).

To understand how the correlator (13) relates to the result of the coherent state formalism (1), the limit of a large number N of emission points is relevant. $T_c(\mathbf{P}_1, \mathbf{P}_2)$ in (13) is a sum over N terms while the other terms in the nominator and denominator are sums of N^2 terms. In this sense, the T_c -dependence of (13) provides a finite multiplicity correction and can be neglected as a subleading $\frac{1}{N}$ -contribution in the large N limit of (13),

$$\lim_{N \rightarrow \infty} C(\mathbf{P}_1, \mathbf{P}_2) = 1 + \frac{e^{-\frac{\sigma^2}{2} \mathbf{Q}^2} \left| \sum_{i=1}^{\infty} s_i(\mathbf{K}) e^{iq_i Q} \right|^2}{\left(\sum_{i=1}^{\infty} s_i(\mathbf{P}_1) \right) \left(\sum_{j=1}^{\infty} s_j(\mathbf{P}_2) \right)}. \quad (14)$$

We note that in the derivation of (1), subleading $\frac{1}{N}$ -contributions are dropped [2]. The large N approximations (1) and (14) are clearly justified for pion interferometry in ultrarelativistic (Pb-Pb) heavy ion collisions where typical pion multiplicities are in the hundreds. For smaller systems, however, and especially in studies of the multiplicity dependence of HBT correlations [15], one might wish to start from the expression for finite multiplicity (13). Expression (14) can be obtained from the coherent state result (1) by inserting

$$S(x, K) = \sum_{i=1}^{\infty} S_i(x, \mathbf{K}), \quad (15a)$$

$$S_i(x, \mathbf{K}) = \mathcal{N} \delta(t - t_i) e^{-\frac{1}{\sigma^2} (\mathbf{x} - \mathbf{q}_i)^2} \times e^{-\sigma^2 (\mathbf{K} - \mathbf{p}_i)^2}, \quad (15b)$$

where \mathcal{N} is an arbitrary normalization factor. In this sense, (15a) is the emission function for a source Σ with initial spatial localization σ . It contains the information about how the *initial* phase space emission points z_i and the *measured* momenta \mathbf{K} are correlated. Spatial and temporal components are not treated equally in (15b), since our derivation is not Lorentz covariant. The Lorentz covariant setting used in (1) allows for an additional dependence of $S(x, K)$ on the temporal component of K which does not exist in our derivation. In practical

applications however, the emission function (1) is used in the so-called on-shell approximation, where this additional K_0 -dependence is not employed, [3].

Both the 2-particle correlator (13) and the 1-particle spectrum $\nu(\mathbf{P})$ in (8b) depend on the initial spatial localization σ which is an additional free parameter. We now discuss this σ -dependence. We first consider the limit $\sigma \rightarrow 0$, in which the Gaussian wavepacket (4a) describes at freeze out ($t = t^{(i)}$) a state with position uncertainty $\Delta \mathbf{x} = 0$, i.e., the source is sharply (“classically”) localized in configuration space. The prize for this optimal spatial information is that nothing can be said about the initial momenta \mathbf{p}_i at emission, the one-particle spectrum $\nu(\mathbf{P})$ is flat. The measured momentum correlations contain spatial information about the source, namely

$$\lim_{\sigma \rightarrow 0} C(\mathbf{P}_1, \mathbf{P}_2) = 1 + \frac{\sum_{(i,j)} \cos((q_i - q_j) \cdot (P_1 - P_2))}{N(N-1)}. \quad (16)$$

Due to the cos-term, the dependence of the 2-particle correlator (16) on the *measured* relative momentum $P_1 - P_2$ gives information on the *initial* relative distances $q_i - q_j$ in the source. This is the HBT effect. Eq. (16) differs significantly from the cos-prescription (2): here, $P_1 - P_2$ is the measured relative pair momentum, while $p_i - p_j$ in (2) denotes the initial momentum difference. As a consequence, the sum $\sum_{(i,j)}$ in (16) goes over *all* pairs irrespective of the momenta p_i, p_j since in the limit $\sigma \rightarrow 0$, all information about these initial momenta is lost, while the sum in (2) goes only over those pairs for which the initial relative pair momentum $p_i - p_j$ lies in the same bin as the measured $P_1 - P_2$. Since the correlator (16) is a limiting case of (14), it is always larger than unity. In contrast, due to the wrong pair selection criterion, the correlator (2) can drop below 1. This insufficiency of (2) becomes more significant for sources with strong q - p position-momentum correlation, as was noticed in [9].

The other limiting case of (13) is the plane wave limit

$$\lim_{\sigma \rightarrow \infty} C(\mathbf{P}_1, \mathbf{P}_2) = 1 + \delta_{\mathbf{P}_1, \mathbf{P}_2}. \quad (17)$$

In this limit, nothing can be said about the spatio-temporal extension of the source since the 2-particle symmetrized wave functions (5) contain no space-time information.

The difference between (16) and (17) shows that the σ -dependence of the 2-particle correlator cannot be neglected. As pointed out already in [10,11], none of the two limits is realistic. For $\sigma \rightarrow 0$, one has sharp information in configuration space but the momentum space information is lost and hence, the set Σ of phase space emission points $(\mathbf{q}_i, \mathbf{p}_i, t_i)$ contains no information about the one-particle momentum spectrum $\nu(\mathbf{P})$. In the limit $\sigma \rightarrow \infty$, on the other hand, no space-time information is contained in Σ . A realistic width σ hence lies in between these two extremes.

For further discussing the σ -dependence of 1- and 2-particle spectra, we now generalize Eq. (15) to continuous phase space distributions $\rho(\mathbf{q}, \mathbf{p}, t)$ which can encode statistical assumptions. To this aim, we extend the sum (15a) to a phase space integral weighted by a (classical) distribution ρ , [2,13]

$$S(x, K) = \int d^3\mathbf{q}_i d^3\mathbf{p}_i dt_i \rho(\mathbf{q}_i, \mathbf{p}_i, t_i) S_i(x, \mathbf{K}). \quad (18)$$

The one-particle spectrum is obtained from (18) via $\int d^4x S(x, K)$. Especially, for a Boltzmann distribution of temperature T , $\rho(\mathbf{q}, \mathbf{p}, t) \propto \exp(-\frac{\mathbf{p}^2}{2mT})$, this one-particle spectrum has an effective temperature [11]

$$T_{\text{eff}} = T + \frac{1}{2m\sigma^2}. \quad (19)$$

Hence, the one-particle spectrum broadens significantly for a narrow spatial width σ . σ is a free parameter which has to be determined from a comparison to data. How can this be done? One idea is to look at systems which can be expected to provide very small, almost pointlike boson emission regions. Candidates are e.g. the $p\bar{p}$ annihilation process [14] or Z_0 -decays [15]. The width of the HBT-correlator determined for these systems should be dominated by the width σ . To obtain an argument supporting this idea, we consider the extreme case of a “pointlike source” ρ with no momentum dependence, for which all particles are emitted from the same space-time position $\tilde{\mathbf{q}}, \tilde{t}$. Calculating the emission function (18) for the corresponding $\rho(\mathbf{q}_i, \mathbf{p}_i, t_i) = \delta^{(3)}(\mathbf{q}_i - \tilde{\mathbf{q}})\delta(\tilde{t} - t_i)$, we find

$$C(\mathbf{P}_1, \mathbf{P}_2) = 1 + e^{-\frac{\sigma^2}{2}\mathbf{Q}^2}, \quad (20a)$$

$$R_{\text{point}}^{\text{HBT}} = \sigma/\sqrt{2}. \quad (20b)$$

Several assumptions enter this result: for pointlike sources with an additional momentum dependence, the correlation is in general more complicated. Also, the width σ could in principle depend on the emission points z_i , the localized wavepackets could have a different, non-Gaussian shape, etc. Still, Eq. (20) suggests that the size of the HBT radius parameters measured for very small boson emitting systems is essentially given by σ .

From the pion interferometric measurements of systems like the $p\bar{p}$ annihilation process or Z_0 -decays [14,15], one infers on the basis of (20b) a pion wavepacket width of the order $\sigma \approx 1$ fm. This is in good agreement with the natural localization scale of the pion, its Compton wavelength [14]. Remarkably, for such a localization, the additional quantum contribution to the temperature T_{eff} in (20) is of order $\frac{1}{2m\sigma^2} \approx 100$ MeV. This indicates that the initial spatial localization width σ plays an important role in accounting for the slope of the measured one-particle spectra.

In the present formalism, the role of an event generator for the boson emitting source is to provide a dynamical

calculation of the phase space occupation Σ from some more fundamental initial condition. The current praxis for event generators of heavy ion collisions amounts to determining the 1-particle spectrum in the limit $\sigma \rightarrow \infty$. We have shown that this limit is unrealistic and that a realistic spatial wavepacket width leads to a substantial broadening of the one-particle spectrum. Our main result is an algorithm which allows for the calculation of both the one-particle spectra via $\nu(\mathbf{P})$ in (8b), and the two-particle correlations via $C(\mathbf{P}_1, \mathbf{P}_2)$ in (13), starting from an arbitrary initial phase space distribution Σ of wavepackets with arbitrary spatial localization σ . Here, the spatial width σ is an additional free parameter which has to be fixed in comparison with experimental data. The slope of the transverse mass spectra (19) provide a lower bound for σ while the 2-particle correlators provide an upper bound. For pions these bounds are very tight and a realistic width σ is of the order of the pion Compton wavelength.

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- [1] E. Shuryak, Phys. Lett. B**44**, 387 (1973).
 - [2] S. Chapman and U. Heinz, Phys. Lett. B**340**, 250 (1994).
 - [3] U. Heinz, Lectures given at the NATO ASI on “Correlations and Clustering Phenomena in Subatomic Physics” at Dronten, Netherlands, nucl-th/9609029.
 - [4] H. Sorge, H. Stöcker and W. Greiner, Ann. Phys. **192**, 266 (1989).
 - [5] K. Werner, Phys. Rep. **232**, 87 (1993).
 - [6] Y. Pang, T.J. Schlagel and S.H. Kahana, Phys. Rev. Lett. **68** (1992) 2743; T.J. Schlagel, Y. Pang and S.H. Kahana, Phys. Rev. Lett. **69** (1992) 3290.
 - [7] J. Aichelin, Nucl. Phys. A (1997) in press (nucl-th/9609006).
 - [8] S. Pratt, T. Csörgö and J. Zimanyi, Phys. Rev. C**42**, 2646 (1990).
 - [9] M. Martin, H. Kalechofsky, P. Foka and U.A. Wiedemann, nucl-th/9612023.
 - [10] S. Padula, M. Gyulassy and S. Gavin, Nucl. Phys. B**329**, 357 (1990).
 - [11] H. Merlitz and D. Pelte, nucl-th/9702005.
 - [12] M. Hillery, R.F. O’Connell, M. Scully and E.P. Wigner, Phys. Rep. **106**, 121 (1984).
 - [13] Q.H. Zhang, U.A. Wiedemann, C. Slotta and U. Heinz, in preparation.
 - [14] G. Goldhaber, S. Goldhaber, W. Lee and A. Pais, Phys. Rev. **120**, 300 (1960).
 - [15] OPAL Collab., G. Alexander et al., CERN-PPE/96-90.